MECHANICS LABORATORY
AM 317

EXPERIMENT 1
BEAM DEFLECTIONS

I. OBJECTIVES

1.1 To observe, evaluate and report on the load deflection relationship of a simply supported beam and a cantilever beam.

1.2 To determine the modulus of elasticity of the beam and what the material the beam is made of using beam deflection theory.

1.3 To verify the principle of superposition and Maxwell's Reciprocity Theorem.

II. INTRODUCTION AND BACKGROUND

The deflections of a beam are an engineering concern as they can create an unstable structure if they are large. People don't want to work in a building in which the floor beams deflect an excessive amount, even though it may be in no danger of failing. Consequently, limits are often placed upon the allowable deflections of a beam, as well as upon the stresses.

When loads are applied to a beam their originally straight axes become curved. Displacements from the initial axes are called bending or flexural deflections. The amount of flexural deflection in a beam is related to the beams area moment of inertia (I), the single applied concentrated load (P), length of the beam (L), the modulus of elasticity (E), and the position of the applied load on the beam. The amount of deflection due to a single concentrated load P, is given by:

$$\delta = \frac{PL^3}{kEI} \qquad 1.1$$

where k is a constant based on the position of the load, and on the end conditions of the beam. For deflection of specific loading conditions refer to Table 1.

The bending stress at any location of a beam section is determined by the flexure formula:

$$\sigma = \frac{My}{I} \qquad 1.2$$

where:

- **M - moment at the section**
- **y - distance from the neutral axis to the point of interest**
- **I - moment of inertia**

The largest stress at the same section follows from this relation (Equation 1.2) by taking y at an extreme fiber at distance c, which leads to:

$$\sigma_{max} = \frac{Mc}{I} \qquad 1.3$$

III. APPARATUS

3.1 Simply-supported and Cantilever rectangular beams

3.2 Weights

3.3 Micrometer

3.4 Ruler

3.5 Dial Gauges

IV PROCEDURE

PART A - SIMPLY SUPPORTED BEAM

4.1A Record the beam dimensions and calculate the area moment of inertia (I) using:

$$I = \frac{bh^3}{12} \qquad 1.4$$

4.2A Calculate the maximum permissible loads for mid-span and quarter-span loading using Equation 1.3, where maximum allowable stress is 18,000 psi.

<u>IMPORTANT</u> - Check these calculated maximum loads with instructor before proceeding with the experiment.

4.3A Load the beam at the mid-span in 5 lb. increments, until the maximum load limit is reached. Record the deflection at the point of loading at each incremental load. Small divisions on the dial gage are 0.001 inch. One full revolution of the dial is 0.1 inch (100 small divisions).

4.4A Repeat the above procedure at the quarter-span.

PART B- CANTILEVER BEAM

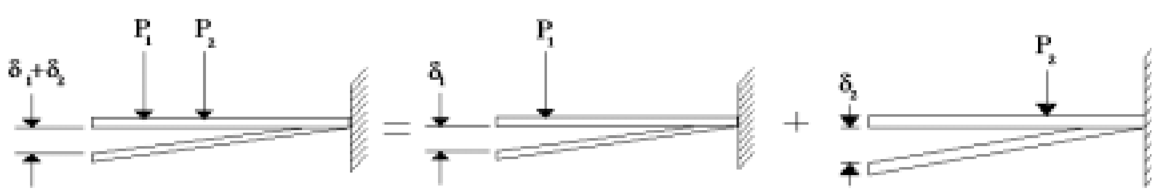
4.1B Record the beam dimensions and calculate the area moment of inertia (I) using Equation 1.4.

4.2B Calculate the safe loads at the mid-span and end of the cantilever beam using Equation 1.3, and a maximum allowable stress of 18,000 psi.

4.3B Load the beam at the mid-span in 2 lb. increments, until the maximum load limit is reached. Record the deflection at the point of loading at each increment.

4.4B Repeat the above procedure at the free end of the beam. Care must be taken so that the displacement does not exceed the maximum travel of the dial gage.

PART C - THE PRINCIPLE OF SUPERPOSITION



4.1C Choose a convenient reference point on either beam. All deflections will be measured at this point.

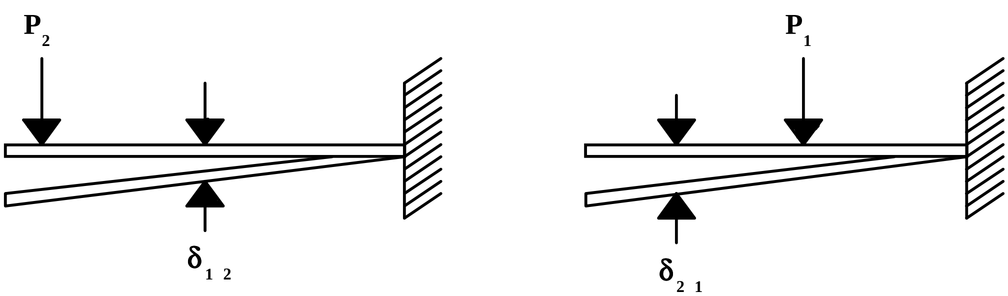
4.2C Place a single concentrated load at some point other than the reference point and measure the resulting deflection (δ1) at the reference point.

4.3C Remove the first load, and place a second load at another point on the beam and measure the resulting deflection (δ2) at the reference point.

4.4C Apply both loads simultaneously and measure the resulting deflection (δ3) at the reference point. The sum of the single deflection (δ3) should closely approximate the total deflection (δ1 + δ2).

$$\delta_3 = \delta_1 + \delta_2 \qquad 1.5$$

PART D - MAXWELL'S RECIPROCITY THEOREM



4.1D Choose two non-symmetrical reference points on either beam.

4.2D Apply a concentrated load (P1) at one point and measure the resulting deflection (δ21) at the other point.

4.3D Remove the load from the first reference point and place a different load (P2) on the second reference point. Measure the resulting deflection (δ12) at the first reference point. The loads and the deflections should satisfy the following relationship:

$$P_1\delta_{12} = P_2\delta_{21} \qquad 1.6$$

V. REPORT

5.1 Plot the curve of load versus deflection for two loading configurations of the simply supported beam. Show loads as ordinates and deflections as abscissas. Include this plot in the Results section with proper title, axis labels and figure number. Raw data goes in the Appendix.

5.2 Plot load versus deflection for the two loading configurations of the cantilever beam. Include this plot in the Results section. Data goes in the Appendix

5.3 Referring to the configuration in Table 1 and selecting the appropriate equations, determine the values of modulus of elasticity for each loading conditions. Create a table of the results you obtained for the modulus of elasticity for the simply supported beam (two load cases) and the cantilever beam (two load cases). Raw data goes in the Appendix.

5.4 Determine what material the beams are made from by comparing the modulus of elasticity you calculated to values referenced in a Materials or Strength of Materials text.

5.6 Verify the validity of the principle of Superposition and Maxwell's Reciprocity theorem by performing all necessary calculations. Present these results in the Results section.

VI. SELECTED REFERENCES

6.1 Statics and Strength Materials, Hibler

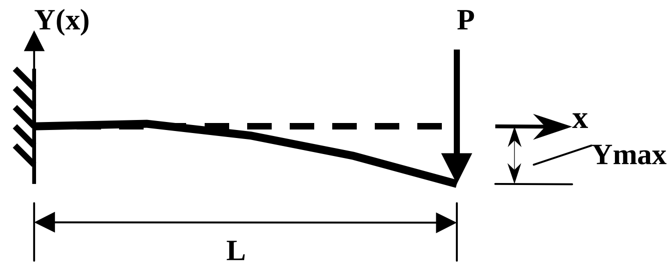
6.2 Introduction to Mechanics of Solids, Popov

6.3 Statics and Strength for Materials, Stevens, Karl K.

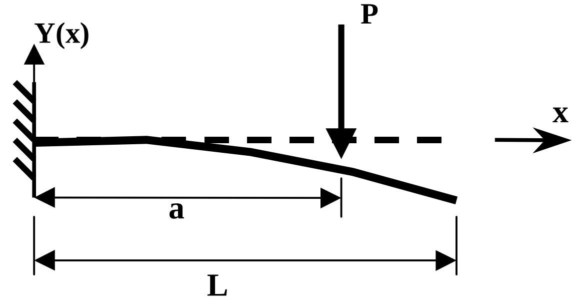
6.4 Design Analysis of Beams and Shafts, Hopkins, R. Bruce

Table 1.
Deflection Equations for Cantilever and Simply-Supported Beams

CANTILEVER BEAM: DEFLECTION Y AS A FUNCTION OF X



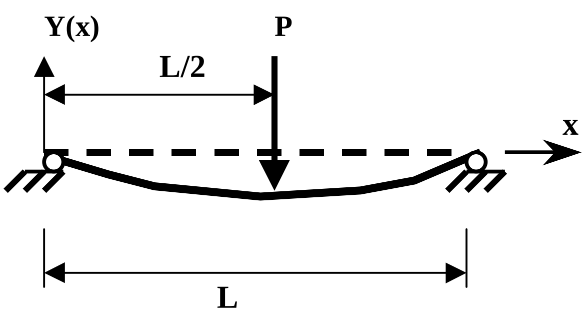
$x = L \qquad Y_{max} = \dfrac{-P \cdot L^3}{3E\,I}$



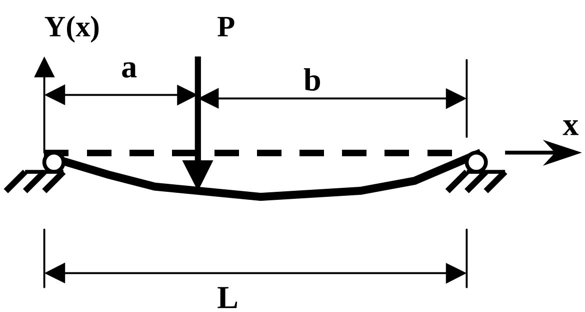
$0 < x \le a \qquad Y(x) = \dfrac{-P\,x^2}{6\,E\,I}\,(3\,a - x)$

$a \le x \le L \qquad Y(x) = \dfrac{-P\,a^2}{6E\,I}\,(3\,x - a)$

SIMPLY-SUPPORTED BEAM: DEFLECTION Y AS A FUNCTION OF X



$x = \dfrac{L}{2} \qquad Y_{max} = \dfrac{-P \cdot L^3}{48E\,I}$

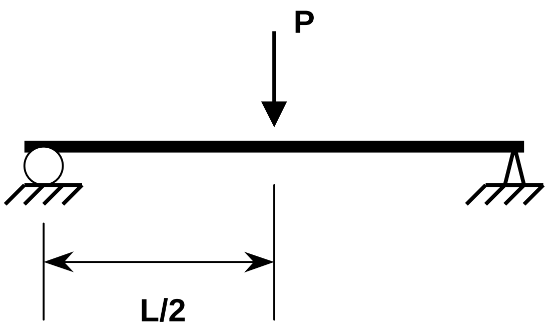


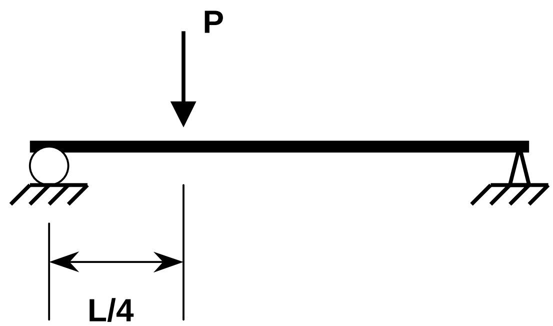
$x \le a \qquad Y(x) = \dfrac{-P\,b\,x}{6E\,I\,L}\left(L^2 - x^2 - b^2\right)$

$x = a = \dfrac{L}{4} \qquad Y = \dfrac{-3P\,L^3}{256E\,I}$

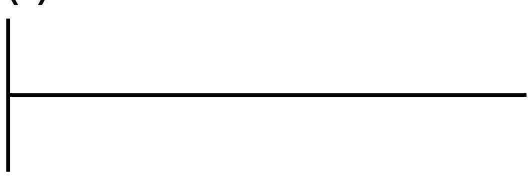
SIMPLY SUPPORTED BEAM DATA	
LENGTH L (INCHES)	
CROSS-SECTION HEIGHT h (INCHES)	
CROSS-SECTION WIDTH b (INCHES)	
MOMENT OF INERTIA I (IN^4)	

TABLE 2
SIMPLY SUPPORTED BEAM DATA

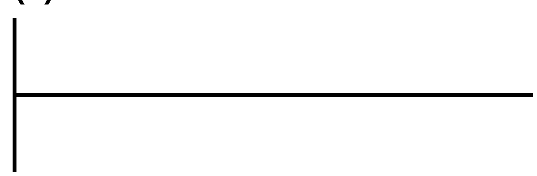




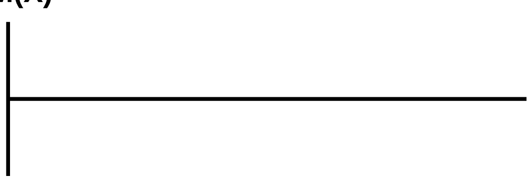
V(X)



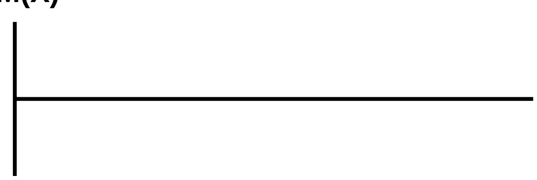
V(X)



M(X)



M(X)

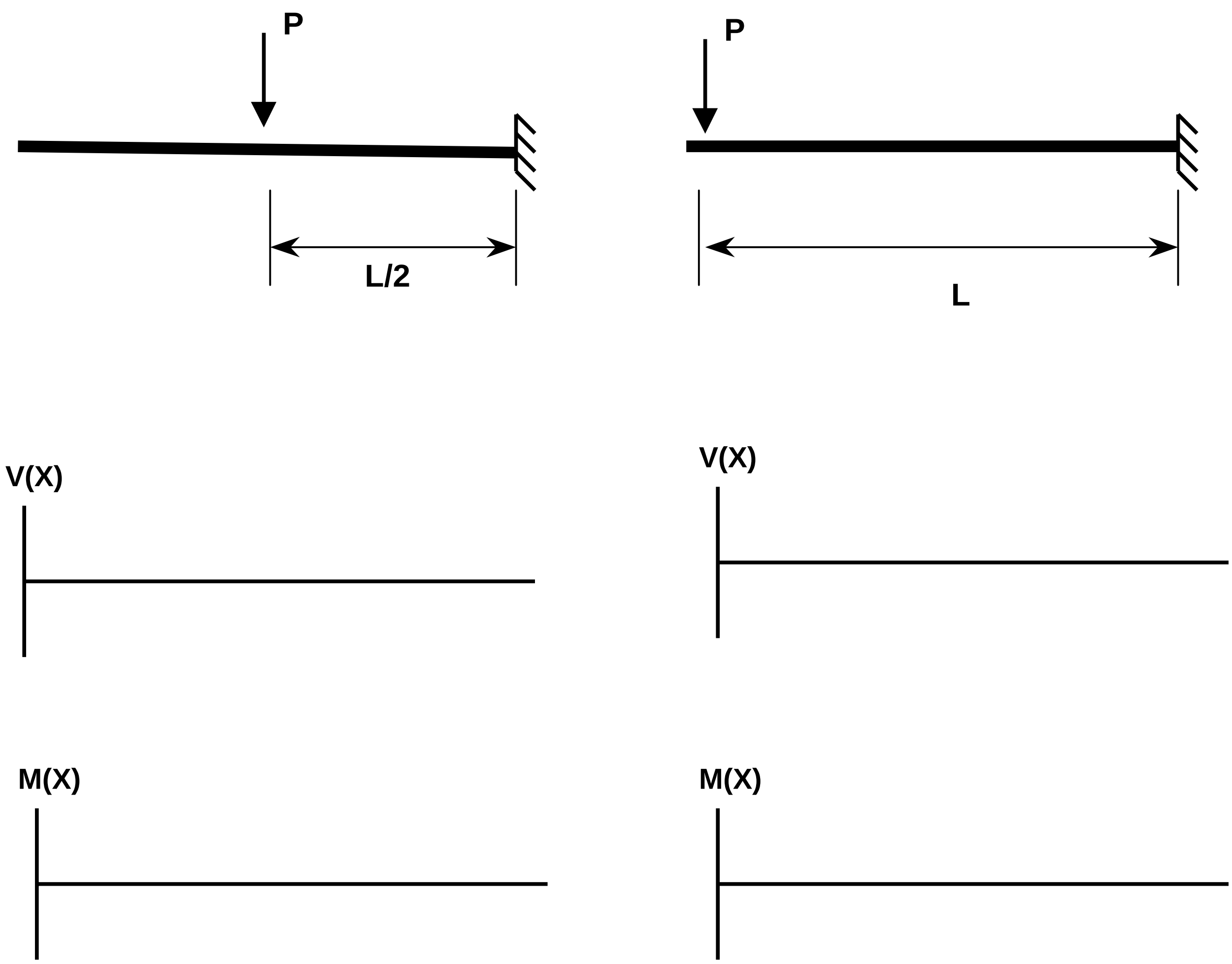


SAFE LOAD =

SAFE LOAD =

CANTILEVER BEAM DATA	
LENGTH L (INCHES)	
CROSS-SECTION HEIGHT h (INCHES)	
CROSS-SECTION WIDTH b (INCHES)	
MOMENT OF INERTIA I (IN^4)	

TABLE 3
CANTILEVER BEAM DATA



SIMPLY SUPPORTED BEAM				
LOAD (LB)	¼ SPAN DEFLECTION	MODULUS OF ELASTICITY E	MID-SPAN DEFLECTION	MODULUS OF ELASTICITY E
5				
10				
15				
20				
25				
30				
35				
40				
45				
50				
	AVERAGE E =		AVERAGE E =	

TABLE 4
SIMPLY SUPPORTED BEAM DATA

CANTILEVER BEAM				
LOAD (LB)	MID-SPAN DEFLECTION	MODULUS OF ELASTICITY E	END DEFLECTION	MODULUS OF ELASTICITY E
2				
4				
6				
8				
10				
12				
14				
16				
18				
20				
22				
24				
26				
28				
	AVERAGE E =		AVERAGE E =	

TABLE 5
CANTILEVER BEAM DATA

BEAM USED:		
POINT	LOAD (LB)	DEFLECTION (IN)
P1		
P2		
BOTH		
EXPERIMENTAL DEFLECTION:		
THEORETICAL DEFLECTION (EQUATIONS TABLE 1):		
% ERROR (REF TO EXPERIMENTAL VALUE):		

TABLE 6
SUPERPOSITION

BEAM USED:		
LOAD (LB)	DEFLECTION (INCH)	
	EXPERIMENTAL	THEORETICAL
$P_1\ \delta_{12}$ =		
$P_2\ \delta_{21}$ =		
% ERROR =		

TABLE 7
MAXWELL'S RECIPROCITY